Theorem 4.3. *Let D be an oriented checkerboard colorable virtual link diagram. Then*

(1) $AS(D)$ only contains even integer; and

(2) for any summand $A^s K_{i_1}^{j_1} K_{i_2}^{j_2} \cdots K_{i_v}^{j_v}$ with $1 \leq i_1 < i_2 < \cdots < i_v$, $j_t \geq 1$ for $t = 1, 2, \cdots, v$, and $v \geq 1$ of $\langle D \rangle_{NA}$, we have $2i_v \leq \sum_{t=1}^{v} i_t \cdot j_t$. In particular, $\langle D \rangle_{NA}$ has no summands like $A^s K_i$.

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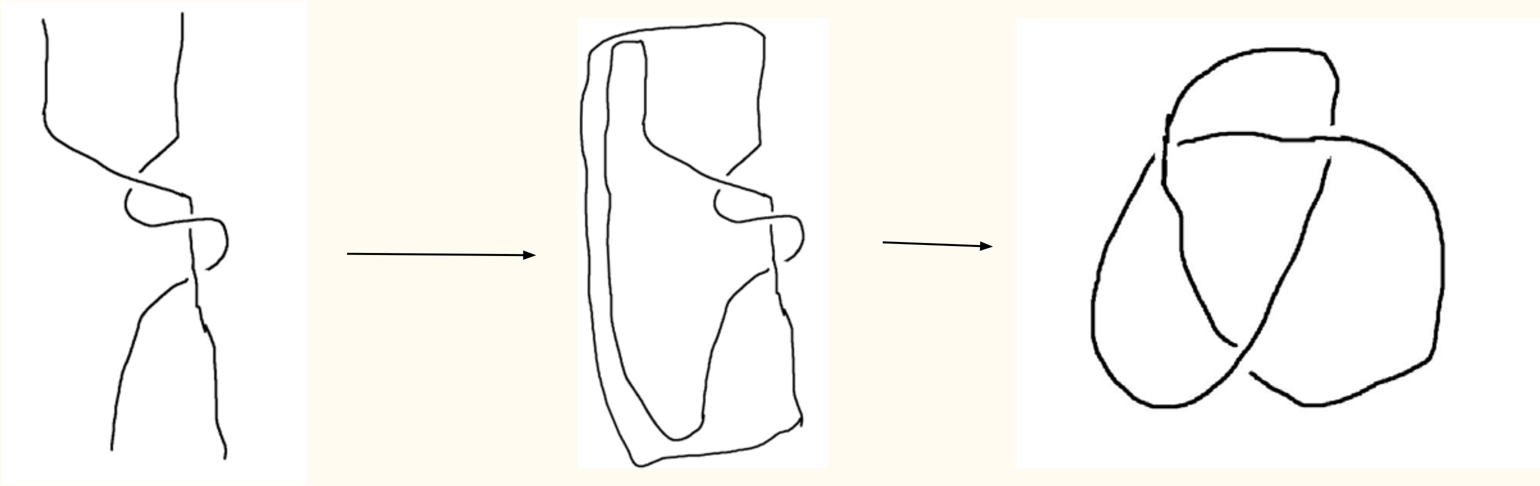
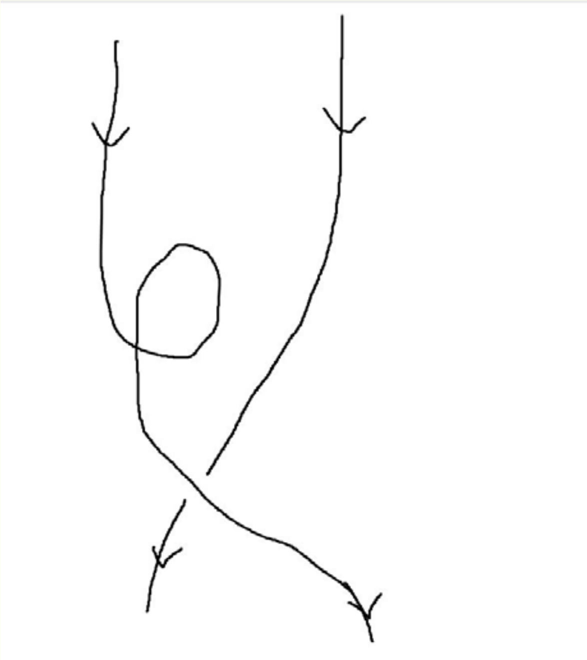
Louis H. Kauffman (2020)

Braids

Alexander's theorem: every knot or link can be represented by a closed braid.z

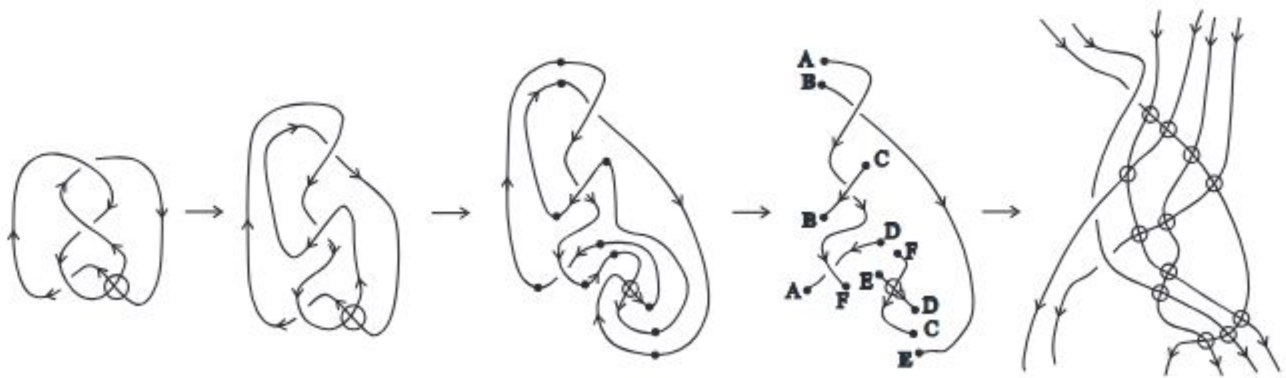


Invalid Braid



Virtual Braids

Theorem 4.1. *([13], Theorem 1) Every (oriented) virtual link can be represented by a virtual braid whose closure is equivalent to the original link.*



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Virtual Braiding Process

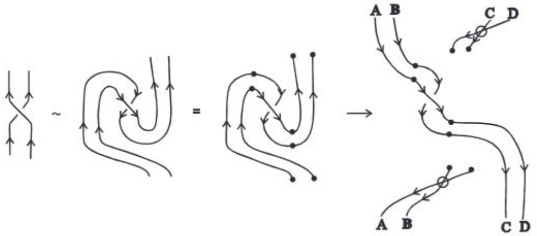

Fig. 12: Full twist for a classical crossing.

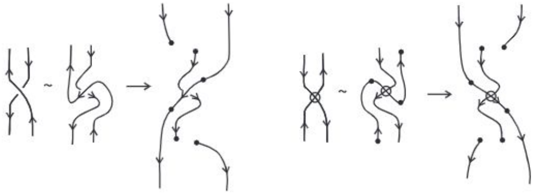

Fig. 13: Half twist for a classical/virtual crossing.

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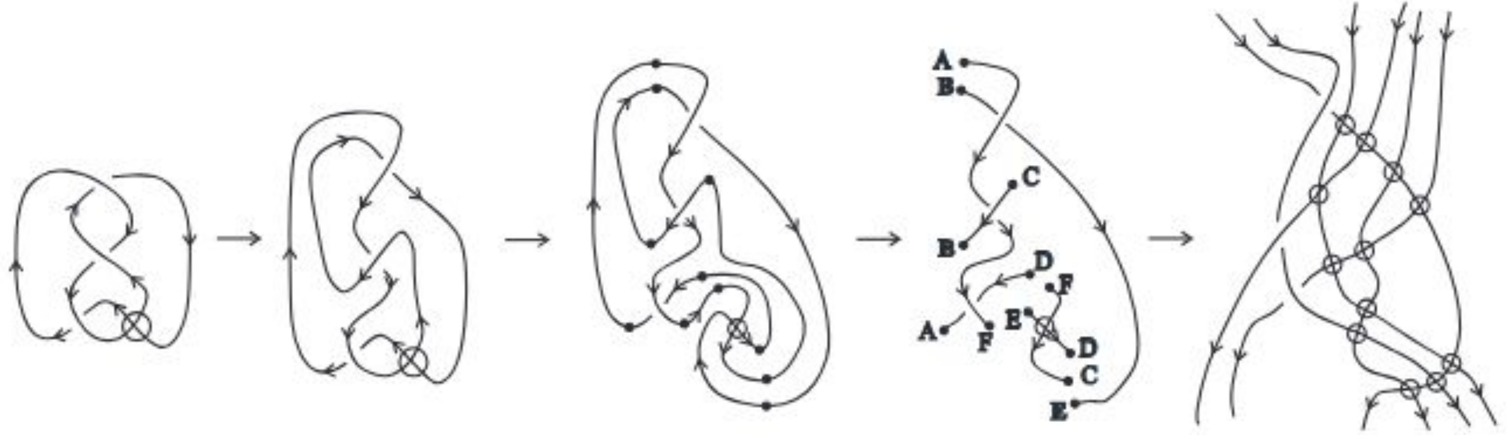
A
B
C
B
D
F
A
F
E
D
C
E

Virtual Braid's and checkerboard colorability

Remark 4.2. *According to the braiding technique, described in Theorem 1 [13], which just changes the relative position of classical and virtual crossings by crossing rotation, the original virtual link diagram and the closure of its virtual braid have the same checkerboard colorability.*

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Application of Theorem 4.3

Theorem 4.3. *Let D be an oriented checkerboard colorable virtual link diagram. Then*

(1) $AS(D)$ only contains even integer; and

(2) for any summand $A^s K_{i_1}^{j_1} K_{i_2}^{j_2} \cdots K_{i_v}^{j_v}$ with $1 \leq i_1 < i_2 < \cdots < i_v$, $j_t \geq 1$ for $t = 1, 2, \cdots, v$, and $v \geq 1$ of $\langle D \rangle_{NA}$, we have $2i_v \leq \sum_{t=1}^{v} i_t \cdot j_t$. In particular, $\langle D \rangle_{NA}$ has no summands like $A^s K_i$.

$$\langle 4.55 \rangle_{NA} = A^4 + A^{-4} + 1 - (A^4 + A^{-4} + 2)K_1^2 + 2K_2,$$

$$\langle 4.56 \rangle_{NA} = A^4\left(-\left(K_1{}^2 - 1\right)\right) + \frac{1 - K_1{}^2}{A^4} - 2K_1{}^2 + 2K_2 + 1,$$

$$\langle 4.59 \rangle_{NA} = \frac{A^8\left(K_2 - K_1{}^2\right) + A^4\left(3 - 2K_1{}^2\right) - K_1{}^2 + K_2}{A^4},$$

$$\langle 4.72 \rangle_{NA} = 1,$$

$$\langle 4.76 \rangle_{NA} = \frac{A^8\left(K_2 - K_1{}^2\right) + A^4\left(3 - 2K_1{}^2\right) - K_1{}^2 + K_2}{A^4},$$

$$\langle 4.77 \rangle_{NA} = \frac{A^8\left(K_2 - K_1{}^2\right) + A^4\left(3 - 2K_1{}^2\right) - K_1{}^2 + K_2}{A^4},$$

$$\langle 4.96 \rangle_{NA} = \frac{K_1{}^2}{A^6} + A^4(K_3 - K_1K_2) - A^2\left(K_1{}^2 - 1\right) - K_1K_2 + K_1.$$

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Virtual Knot 4.72